A.P. Calculus –Summer Packet

Going into AP calculus, there are certain skills that have been taught to you over the previous years that I assume you have. If you do not have these skills, you will find that you will consistently get problems incorrect next year, even though you understand the calculus concepts. It is frustrating for students when they are tripped up by the algebra and not the calculus. This summer packet is intended for you to brush up and possibly relearn these topics.

I am assuming that you have basic skills in algebra. Being able to solve equations, work with algebraic expressions, and basic factoring, for example should now be a part of your basic knowledge. If not, you would not be going onto AP calculus. So, only the topics that students consistently do not have down in their basic skill set are included here. These are skills that are used continually in A.P. Calculus.

On the following pages, you have 5 to 12 problems per page. Each problem should be done in the space provided. Rather than give you a textbook to remind you of the techniques necessary to solve the problem, I have given you websites that have full instructions on the techniques. If and when you are unsure of how to attempt these problems, examine these websites. Don't fake your way through these problems. As stated, students are notoriously weak in them, even students who have achieved well prior to AP Calculus. Use the websites.

Realize also that certain concepts are interrelated. Domain, for example, may require you to be an expert at working with inequalities. Solving quadratic equations may involve techniques used in solving fractional equations.

There will be an exam on the material in this packet during the first week of school.   Do not rely on the calculator. Half of your AP exam next year is taken without the calculator. So concentrate on using paper and pencil techniques as much as possible and using the calculator to check your answers.

It is a mistake to decide to do this now. Let it go until mid-summer. I want these techniques to be relatively fresh in your mind in the fall. Also, do not wait to do them at the very last minute. These take time. If you do two concepts a day, the whole packet will take you about a week to complete.

Feel free to email me (nspano@oxfordasd.org) at any point over the summer if you have questions on the summer packet or anything else AP Calculus related. I will most likely send out a group email in late July/early August to set up a summer help session day.   I am excited about next year and look forward to seeing you in the fall.

Mr. Spano

You can certainly do Google searches for any of the topics in this packet. However, here a few sites that will cover pretty much all of these topics if you need help.

Here is a good site for most topics with written explanations:
http://www.purplemath.com/modules/index.htm

Here are two good sites for most topics with video explanations:
http://www.khanacademy.org/
http://patrickjmt.com/

Here is a good site for a review of limits:
http://www.calculus-help.com/tutorials/   Chapter 1 – Lessons 1 through 4

Beginning algebra topics
1. Slope/Equations of Lines
2. Negative and fractional exponents

Intermediate algebra topics
3. Domain
4. Solving inequalities: absolute value
5. Solving inequalities: quadratic
6. Special factoring formulas
7. Logarithms
8. Even and odd functions
9. Solving quadratic equations and quadratic formula

Advanced algebra topics
10. Asymptotes
11. Complex fractions
12. Composition of functions
13. Solving rational (fractional) equations

Trig topics
14. Basic right angle trig
15. More trig topics
16. Trig equations

Introductory calculus topic

17. Limits

Topic 1: Slope and Equations of a Line

Find the slope of the line containing the given points

1. (3, 5) (7, 11)   2. (-8, 4) (-2, -3)   3. (5, 2) (7, 2)   4. (6, -3) (6, -3)

Write the equation of the line with the given conditions in point-slope, and slope-intercept forms.

5. Slope of 3, contains (1, -2)   6. Contains (5, -1) and (-5, 5)

7. Contains (3, 7) and (6, 5)   8. Contains (-4, 2) and (-3, 6)

9. Contains (2,1), parallel to $4x - 2y = 3$ 10. Contains (2,1), perpendicular to $4x - 2y = 3$

Topic 2: Fractional and Negative Exponents

Simplify using only positive exponents and by rewriting any radicals as fractional exponents.

1. $3x^{-3}$

2. $\dfrac{5}{\sqrt{x+4}}$

3. $-6\left(\dfrac{3}{2}\right)(4-9x)^{-\frac{1}{2}}$

4. $\left(16x^2y\right)^{\frac{3}{4}}$

5. $-\dfrac{x^{-\frac{1}{2}}}{2}\sin\sqrt{x}$

6. $2\left(\dfrac{2}{2-x}\right)\left[\dfrac{-2}{(2-x)^2}\right]$

7. $\left(\dfrac{6x^4y^8}{8x^2y^{11}}\right)^{-3}$

8. $\dfrac{\dfrac{1}{2}(2x+5)^{-\frac{3}{2}}}{\dfrac{3}{2}}$

Topic 3: Domain

Find the domain of the following functions.

1. $y = \dfrac{3x-2}{4x+1}$

2. $y = \dfrac{x^2-4}{2x+4}$

3. $y = \dfrac{x^2-5x-6}{x^2-3x-18}$

4. $y = \dfrac{2^{2-x}}{x}$

5. $y = \sqrt{x-3} - \sqrt{x+3}$

6. $y = \dfrac{\sqrt{2x-9}}{2x+9}$

7. $y = \dfrac{x^2+8x+12}{\sqrt{x+5}}$

8. $y = \sqrt{x^2-5x-14}$

9. $y = \dfrac{\sqrt[3]{x-6}}{\sqrt{x^2-x-30}}$

10. $y = \ln(2x-12)$

11. $y = \dfrac{x}{\cos x}$

Topic 4: Solving absolute-value inequalities

Write the following absolute value expressions as piecewise expressions.

1. $y = |2x - 4|$

   2. $y = |6 - 5x|$

Solve the following absolute value inequalities.

3. $|x - 3| > 12$

   4. $|x - 3| \leq 4$

   5. $|10x + 8| > 2$

6. $|3x - 4| > -2$

   7. $|x - 6| < -8$

Topic 5: Solving Quadratic Inequalities

Write the following absolute value expressions as piecewise expressions.

1. $\left|x^2 - 1\right|$
2. $\left|x^2 + x - 12\right|$

Solve the following by factoring and making appropriate sign charts.

3. $x^2 - 16 > 0$
4. $x^2 + 6x - 16 > 0$
5. $x^2 - 3x \geq 10$

6. $2x^2 + 4x \leq 3$
7. $2\sin^2 x \geq \sin x \quad 0 \leq x < 2\pi$

Topic 6: Special Factorization

Factor completely.

1. $x^3 + 8$

2. $x^3 - 8$

3. $27x^3 - 125y^3$

4. $x^4 + 11x^2 - 80$

5. $ac + cd - ab - bd$

6. $2x^2 - 20xy + 50y^2$

7. $x^3 - xy^2 + x^2y - y^3$

8. $(x-3)^2 (2x+1)^3 + (x-3)^3 (2x+1)^2$

Topic 7: Logarithms

Use the properties of logarithms to rewrite each problem as a single logarithm.

1. $\ln 2 + \ln x$

2. $\ln x - \ln 6$

3. $4[\ln x + \ln(x+5)] - 2\ln(x-5)$

Use the properties of logarithms to expand the expression as a sum, difference, and/or constant multiple of logarithms.

4. $\ln(5x^2)$

5. $\ln \dfrac{x-7}{2x-1}$

6. $\ln \sqrt{x^2(x+2)}$

Rewrite each exponential as a logarithm.

7. $2^3 = 8$

8. $81^{1/2} = 9$

9. $e^0 = 1$

Rewrite each logarithm as an exponential.

10. $\log_4 16 = 2$

11. $\log_{16} 2 = \dfrac{1}{4}$

12. $\ln e = 1$

Solve each logarithmic or exponential equation.

13. $-14 + 3e^x = 11$

14. $5 + 2\ln x = 4$

15. $-2 + 2\ln 3x = 17$

Topic 8: Even and Odd Functions

Show work to determine if the relation is even, odd, or neither.

1. $f(x) = 2x^2 - 7$ 2. $f(x) = -4x^3 - 2x$ 3. $f(x) = 4x^2 - 4x + 4$

4. $f(x) = x - \dfrac{1}{x}$ 5. $5x^2 - 6y = 1$ 6. $f(x) = e^x - \dfrac{1}{e^x}$

7. $f(x) = 2x\sqrt{x^2 + 3}$ 8. $y = |3x|$

Topic 9: Solving Quadratic Equations and the Quadratic Formula

Solve each equation.

1. $7x^2 - 3x = 0$

2. $4x(x-2) - 5x(x-1) = 2$

3. $x^2 + 6x + 4 = 0$

4. $2x^2 - 3x + 3 = 0$

5. $2x^2 - (x+2)(x-3) = 12$

6. $x + \dfrac{1}{x} = \dfrac{13}{6}$

7. $x^4 - 9x^2 + 8 = 0$

8. $x - 10\sqrt{x} + 9 = 0$

9. $\dfrac{1}{x^2} - \dfrac{1}{x} = 6$

Topic 10: Asymptotes

For each function, find the location of any holes and the equations of both the vertical and horizontal asymptotes (if they exist).

1. $y = \dfrac{x}{x-3}$

2. $y = \dfrac{x+4}{x^2-1}$

3. $y = \dfrac{x+4}{x^2+1}$

4. $y = \dfrac{x^2-2x+1}{x^2-3x-4}$

5. $y = \dfrac{x^2-9}{x^3+3x^2-18x}$

6. $y = \dfrac{2x^2+6x}{x^3-3x^2-4x}$

7. $y = \dfrac{x^2-x-6}{x^3-x^2+x-6}$

8. $y = \dfrac{2x^3}{x^3-1}$

9. $y = \dfrac{\sqrt{4x^2}}{2x+5}$

Topic 11: Complex Fractions

Simplify the following:

1. $\dfrac{x}{x-\dfrac{1}{2}}$

2. $\dfrac{\dfrac{1}{x}+4}{\dfrac{1}{x}-2}$

3. $\dfrac{x-\dfrac{1}{x}}{x+\dfrac{1}{x}}$

4. $\dfrac{\dfrac{3}{x}-\dfrac{4}{y}}{\dfrac{4}{x}-\dfrac{3}{y}}$

5. $\dfrac{1-\dfrac{2}{3x}}{x-\dfrac{4}{9x}}$

6. $\dfrac{\dfrac{x^2-y^2}{xy}}{\dfrac{x+y}{y}}$

Topic 12: Composition of Functions

If $f(x) = x^2$, $g(x) = 2x - 1$, $h(x) = 2^x$, find the following:

1. $f(g(2))$

2. $g(f(2))$

3. $f(h(-1))$

4. $h(f(-1))$

5. $g\left(f\left(h\left(\dfrac{1}{2}\right)\right)\right)$

6. $f(g(x))$

7. $g(f(x))$

8. $g(g(x))$

9. $f(h(x))$

Topic 13: Solving Rational Equations

1. $\dfrac{2}{3} - \dfrac{5}{6} = \dfrac{1}{x}$

2. $x + \dfrac{6}{x} = 5$

3. $\dfrac{x+1}{3} - \dfrac{x-1}{2} = 1$

4. $\dfrac{x-5}{x+1} = \dfrac{3}{5}$

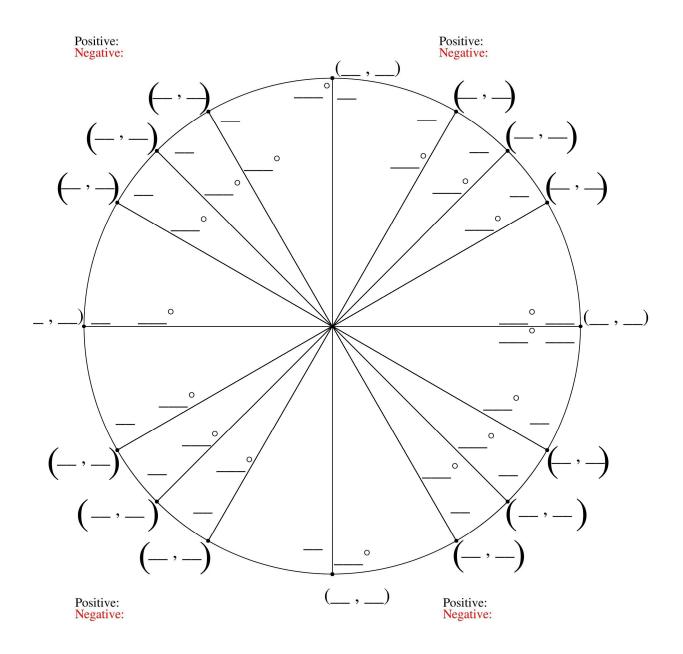5. $\dfrac{60}{x} - \dfrac{60}{x-5} = \dfrac{2}{x}$

6. $\dfrac{2}{x+5} + \dfrac{1}{x-5} = \dfrac{16}{x^2 - 25}$

7. $\dfrac{2x+3}{x-1} = \dfrac{10}{x^2-1} + \dfrac{2x-3}{x+1}$

8. $\dfrac{x}{2x-6} - \dfrac{3}{x^2-6x+9} = \dfrac{x-2}{3x-9}$

Unit Circle Practice

Practice filling out (or creating your own) unit circle until you can do it in under 5 minutes.

Positive:
Negative:

Positive:
Negative:

$(\_\_ , \_\_)$

$(\_ , \_)$

$(\_ , \_)$

$(\_ , \_)$

$(\_ , \_)$

$(\_ , \_)$

$(\_ , \_)$

$\_ , \_)$

$(\_\_ , \_\_)$

$(\_ , \_)$

$(\_ , \_)$

$(\_ , \_)$

$(\_ , \_)$

$(\_ , \_)$

$(\_ , \_)$

$(\_\_ , \_\_)$

Positive:
Negative:

Positive:
Negative:

Topic 14: Basic Trigonometry

Convert each angle from degrees to radians

1. 90°   2. 60°   3. 210°   4. 315°   5. 270°

Convert each angle from degrees to radians

6. π 7. $\dfrac{3\pi}{4}$ 8. $\dfrac{4\pi}{3}$ 9. $\dfrac{11\pi}{6}$ 10. $\dfrac{\pi}{6}$

Find the **exact** value (No calculator!!!)

11. $\sin\dfrac{3\pi}{4}$ 12. $\cos\dfrac{\pi}{6}$ 13. $\tan\dfrac{\pi}{2}$

14. $\csc\dfrac{7\pi}{6}$ 15. $\sec\pi$ 16. $\cot\dfrac{5\pi}{3}$

Topic 15: More Trigonometry Topics

For 1 and 2, point P is on the terminal side of θ, find all 6 trig functions of θ. Draw a picture.

1. $P(-2,4)$ 2. $P(\sqrt{5},-2)$

3. If $\cos\theta = -\dfrac{5}{13}$, and θ in quadrant II, find $\sin\theta$ and $\tan\theta$.

4. If $\cot\theta = 3$, and θ in quadrant III, find $\sin\theta$ and $\cos\theta$.

Find the exact value of the following without a calculator:

5. $\sin^2\left(\dfrac{5\pi}{4}\right) - \cos^2\left(\dfrac{5\pi}{3}\right)$

6. $\left(6\sec\pi - 4\cot\dfrac{\pi}{2}\right)^2$

7. $\left(4\cos\dfrac{\pi}{6} - 6\sin\dfrac{2\pi}{3}\right)^{-2}$

Solve the following triangles (3 decimal place accuracy)

	A =	a = 21.7 ft	A =	a = 6 ft
8.	B = 16°	b =	9. B =	b =
	C = 90°	c =	C = 90°	c = 95 in

Topic 16: Solving Trigonometric Equations

Solve each equation on the interval $[0, 2\pi)$.

1. $\sin x = \dfrac{1}{2}$

2. $\cos^2 x = \cos x$

3. $2\cos x + \sqrt{3} = 0$

4. $4\sin^2 x = 1$

5. $2\sin^2 x + \sin x = 1$

6. $\cos^2 x + 2\cos x = 3$

7. $2\sin x \cos x + \sin x = 0$

8. $8\cos^2 x - 2\cos x = 1$

9. $\sin^2 x - \cos^2 x = 0$

Topic 17: Limits

Find each limit. If the limit does not exist, state why.

1. $\lim\limits_{x \to -2} \dfrac{2x+1}{3x-4}$

2. $\lim\limits_{x \to 2} \left(2x^3 + 5x^2 + 2x + 1\right)$

3. $\lim\limits_{x \to 5} 36$

4. $\lim\limits_{x \to 5} \dfrac{x^2 - 3x - 10}{x-5}$

5. $\lim\limits_{x \to -2} \dfrac{x^2 - 4}{x-2}$

6. $\lim\limits_{x \to -2} \dfrac{x^3 + 8}{x+2}$

7. $\lim\limits_{x \to 2} \dfrac{x-2}{x^3 - 2x^2 + 2x - 4}$

8. $\lim\limits_{x \to 25} \dfrac{\sqrt{x} - 5}{x - 25}$

9. $\lim\limits_{x \to 0} \dfrac{\sqrt{x+9} - 3}{x}$

10. $\lim\limits_{x \to 2} \begin{cases} x-1, & x \le 2 \\ 2x-3, & x > 2 \end{cases}$

11. $\lim\limits_{x \to 6} \begin{cases} 2x-5, & x \ge 6 \\ x^2 - 3x - 10, & x < 6 \end{cases}$

12. $\lim\limits_{x \to \infty} \dfrac{x+2}{x^2 - 3x + 5}$

13. $\lim\limits_{x \to \infty} \dfrac{3x^2 + 7x + 9}{5x^2 - 4x + 11}$

14. $\lim\limits_{x \to \infty} \dfrac{x^2 + 7x - 9}{2x - 3}$

15. $\lim\limits_{x \to 5} \dfrac{x}{(x-5)^2}$